

Decide if the function is an exponential function. If it is, state the initial value and the base.

1) $y = -9.4 \cdot 6^x$

exponential growth / base = 6 initial value = -9.4

Compute the exact value of the function for the given x-value without using a calculator.

2) $f(x) = \left(\frac{1}{4}\right)^x$ for $x = 3$

$$\left(\frac{1}{4}\right)^3 = \frac{1}{4^3} = \frac{1}{4 \cdot 4 \cdot 4} = \frac{1}{64}$$

3) $f(x) = 5^x$ for $x = -2$

$$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$$

Determine a formula for the exponential function.

4)

x	f(x)
-2	80
-1	40
0	20
1	10
2	5

$$y = ab^x$$

$$y = 20 \left(\frac{1}{2}\right)^x$$

Describe the transformation of $f(x)$ from $g(x)$.

5) $f(x) = 3^{x-1} - 3$; relative to $g(x) = 3^x$

$$= 3^{x-1} - 3$$

right 1 | down 3

State whether the function is an exponential growth function or exponential decay function, and describe its end behavior using limits.

6) $f(x) = 0.7^x$



$$\lim_{x \rightarrow -\infty} f(x) = \infty$$

$$\lim_{x \rightarrow +\infty} f(x) = 0$$

Decide whether the function is an exponential growth or exponential decay function and find the constant percentage rate of growth or decay.

7) $f(x) = 87 \cdot 0.04^x$

exponential decay

% rate: $1 - .04 = .96 \rightarrow -96\%$

8) $f(x) = 8.4 \cdot 1.04^x$

growth

% rate: $.04 \rightarrow 4\%$

Find the exponential function that satisfies the given conditions.

9) Initial value = 34, increasing at a rate of 13% per year

$$y = 34(1.13)^x$$

Evaluate the logarithm.

10) $\log_4 256 = y$

$$4^1 = 4$$

$$4^2 = 16$$

$$4^3 = 64$$

$$4^4 = 256$$

($y=4$) $4^y = 256$

11) $\log_6 \left(\frac{1}{36}\right) = y$

$$6^y = \frac{1}{36} \rightarrow 6^y = \frac{1}{6^2} \rightarrow 6^y = 6^{-2}$$

($y=-2$)

Simplify the expression.

12) $\log_7 7^3$

$$\log_7 7^3 = 3$$

13) $10 \log 16$

$$10^{\log 16} = 16$$

Solve the equation by changing it to exponential form.

$$14) \log_9 x = 4$$

$$9^4 = x$$

$$15) \log x = 2.7$$

$$10^{2.7} = x$$

Find the logistic function that satisfies the given conditions.

$$16) \text{Initial value } = 10, \text{ limit to growth } = 60, \text{ passing through } (1, 20)$$

$$y = \frac{m}{1+Ab^x}$$

$$10 = \frac{60}{1+A b^0}$$

$$1+A=6$$
$$A=5$$

cm

$$y = \frac{60}{1+5b^x}$$

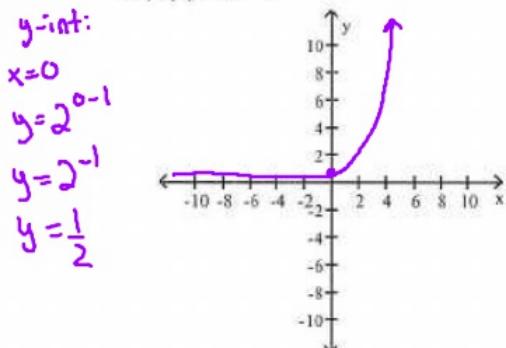
$$20 = \frac{60}{1+5b}$$

$$\begin{aligned} 1+5b &= 3 \\ -1 & \\ 5b &= 2 \\ b &= \frac{2}{5} \end{aligned}$$

$$\boxed{y = \frac{60}{1+5\left(\frac{2}{5}\right)^x}}$$

Sketch the graph of the function.

$$17) f(x) = 2^x - 1$$



$$18) f(x) = \log_2 x$$

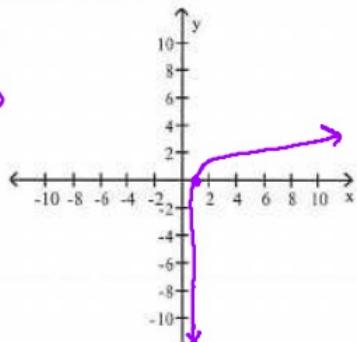
VA: $x=0$

x-int: $y=0$

$$0 = \log_2 x$$

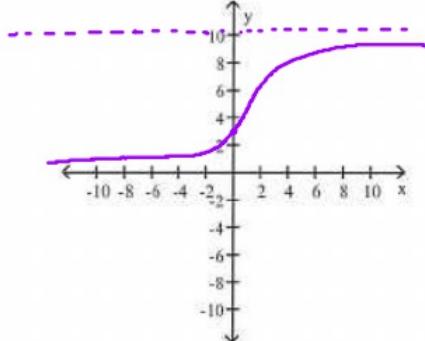
$$2^0 = x$$

$$\boxed{1=x}$$



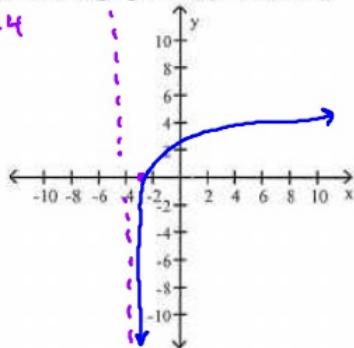
$$19) f(x) = \frac{10}{1 + 2 \cdot 0.4x}$$

$$\text{Max} > 10 \quad y\text{-int: } \frac{10}{1 + 2(0.4)} = \frac{10}{3} = 3\frac{1}{3}$$



20) Sketch a graph of $f(x) = \ln(x+4)$

VA: $x = -4$



$$x\text{-int: } 0 = \ln(x+4)$$

$$e^0 = x+4$$

$$1 = x+4$$

$$-3 = x$$

Describe how to transform the graph of the basic function $g(x)$ into the graph of the given function $f(x)$.

$$21) f(x) = \ln(x+5) - 8; \quad g(x) = \ln x$$

left + down 8

Assuming all variables are positive, use properties of logarithms to write the expression as a sum or difference of logarithms or multiples of logarithms.

$$22) \log_{10}(xy) = \log_{10}(xy) = \log(x) + \log(y)$$

$$23) \log_5\left(\frac{x^7y^9}{2}\right) = \log_5\left(\frac{x^7y^9}{2}\right) = \log_5 x^7 + \log_5 y^9 - \log_5 2 \\ = 7\log_5(x) + 9\log_5(y) - \log_5(2)$$

Use the product, quotient, and power rules of logarithms to rewrite the expression as a single logarithm. Assume that all variables represent positive real numbers.

24) $\log_4 13 - \log_4 a$

$$\log_4 \left(\frac{13}{a} \right)$$

25) $5\log x + 4\log y$

$$\log x^5 + \log y^4 = \log(x^5 y^4)$$

Find the exact solution to the equation.

26) $\log_{10}(x-3) = -1$

$$\begin{aligned} \log_{10}(x-3) &= -1 \\ 10^{-1} &= x-3 \\ \frac{1}{10} &= x-3 \\ +3 & \\ \hline 3\frac{1}{10} &= x \\ \frac{31}{10} &= x \end{aligned}$$

27) $\frac{9 \ln(x-5)}{9} = 1$

$$\begin{aligned} \ln_e(x-5) &= \frac{1}{9} \\ e^{1/9} &= x-5 \end{aligned}$$

28) $9^{7x} = 81$

$$\begin{aligned} 9^{7x} &= 81 \\ 9^{7x} &= 9^2 \\ \frac{7x}{7} &= \frac{2}{7} \\ x &= \frac{2}{7} \end{aligned}$$

29) $100 \left(\frac{1}{5} \right)^{x/2} = \frac{4}{100}$

$$\begin{aligned} \left(\frac{1}{5} \right)^{x/2} &= \frac{1}{25} \\ \left(\frac{1}{5} \right)^{x/2} &= \left(\frac{1}{5} \right)^2 \\ \frac{x}{2} &= 2 \\ x &= 4 \end{aligned}$$

Solve the equation.

$$30) \log(2x) = \log 5 + \log(x-2)$$

$$\log(2x) = \log[5(x-2)]$$

$$2x = 5(x-2)$$

$$\begin{array}{r} 2x = 5x - 10 \\ -5x \quad -5x \\ \hline -3x = -10 \end{array}$$

$$\frac{-3x}{-3} = \frac{-10}{-3} \quad x = \frac{10}{3}$$

$$31) \log(4+x) - \log(x-3) = \log 4$$

$$\log\left(\frac{4+x}{x-3}\right) = \log 4$$

$$(x-3) \frac{4+x}{x-3} = 4(x-3)$$

$$\begin{array}{r} 4+x = 4x-12 \\ -x \quad -x \\ \hline 4 = 3x-12 \\ +12 \quad +12 \\ \hline 16 = 3x \end{array}$$

$$x = \frac{16}{3}$$

$$32) \frac{1000}{1+99e^{-0.3t}} = 250$$

$$\frac{1000}{250} = 1 + 99e^{-0.3t}$$

$$\begin{array}{r} 4 = 1 + 99e^{-0.3t} \\ -1 \quad -1 \\ \hline \frac{3}{99} = \frac{99e^{-0.3t}}{99} \end{array}$$

$$\frac{1}{33} = e^{-0.3t}$$

$$\ln\left(\frac{1}{33}\right) = \ln(e^{-0.3t})$$

$$\frac{\ln\left(\frac{1}{33}\right)}{-0.3} = \frac{-0.3t}{-0.3}$$

$$\boxed{\frac{\ln\left(\frac{1}{33}\right)}{-0.3} = t}$$

Use a calculator to find an approximate solution to the equation.

$$33) 2^x = 17$$

$$\begin{aligned} 2^x &= 17 & x &= \frac{\ln(17)}{\ln(2)} \\ \ln 2^x &= \ln 17 & & \\ \frac{x \ln 2}{\ln 2} &= \frac{\ln 17}{\ln 2} & x &= 4.087 \end{aligned}$$

$$34) e^{-0.15t} = 0.22$$

$$\begin{aligned} e^{-0.15t} &= .22 & t &= 10.094 \\ \ln e^{-0.15t} &= \ln .22 & & \\ -.15t (\ln e) &= \ln (.22) & & \\ \frac{-0.15t}{-.15} &= \frac{\ln (.22)}{-0.15} & & \end{aligned}$$

$$35) 6 \ln(x + 2.8) = 9.6$$

$$\begin{aligned} \frac{6 \ln(x+2.8)}{6} &= \frac{9.6}{6} & e^{1.6} &= x + 2.8 \\ \ln_e(x+2.8) &= 1.6 & e^{1.6} - 2.8 &= x \\ x &= 2.153 & & \end{aligned}$$

Solve the problem.

- 36) Suppose the amount of a radioactive element remaining in a sample of 100 milligrams after x years can be described by $A(x) = 100e^{-0.01022x}$. How much is remaining after 41 years? Round the answer to the nearest hundredth of a milligram.

$$A(41) = 100e^{-0.01022(41)}$$

$$A(41) = 65.77$$

- 37) There are currently 80 million cars in a certain country, increasing by 7.1% annually.
- Write an exponential function that models the situation.
 - How many years will it take for this country to have 94 million cars? Solve algebraically and round to the nearest year.

$$y = 80(1.071)^t$$

$$\frac{94}{80} = \frac{80(1.071)^t}{80}$$

$$1.175 = (1.071)^t$$

$$\ln(1.175) = \ln(1.071)^t$$

$$\frac{\ln(1.175)}{\ln(1.071)} = t \quad t = 2.351$$

- 38) The number of students infected with the flu on a college campus after t days is modeled by the function

$P(t) = \frac{120}{1 + 19e^{-0.4t}}$. What was the initial number of infected students?

$$P(0) = \frac{120}{1 + 19} = \frac{120}{20} = 6$$

- 39) The number of students infected with the flu on a college campus after t days is modeled by the function

$P(t) = \frac{120}{1 + 19e^{-0.4t}}$. What is the maximum number of infected students possible?

120

- 40) The number of students infected with the flu on a college campus after t days is modeled by the function

$P(t) = \frac{120}{1 + 19e^{-0.4t}}$. When will the number of infected students be 100?

$$100 = \frac{120}{1 + 19e^{-0.4t}}$$

$$1 + 19e^{-0.4t} = \frac{120}{100}$$

$$-1 \qquad \qquad -1$$

$$\frac{19e^{-0.4t}}{19} = \frac{2}{19}$$

$$e^{-0.4t} = \frac{1}{95}$$

$$\ln e^{-0.4t} = \ln\left(\frac{1}{95}\right)$$

$$\frac{-0.4t}{-0.4} = \frac{\ln\left(\frac{1}{95}\right)}{-0.4}$$

$$\therefore t = 11.384 \text{ days}$$

